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**System Design Issues for Wireless Communication in a  
Multi-Processor Computer: Carrier Acquisition, Phase  
Noise, and Modulation Constellation**

**by Richard J. Kozick and Brian M. Sadler**

**ARL-TR-2987**

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## **System Design Issues for Wireless Communication in a Multi-Processor Computer: Carrier Acquisition, Phase Noise, and Modulation Constellation**

**Richard J. Kozick**

**Bucknell University, Electrical Engineering Department**

**Brian M. Sadler**

**Computational and Information Sciences Directorate, ARL**

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			Brian M. Sadler	
			19b. TELEPHONE NUMBER (Include area code)	
			(301) 394-1239	

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## 1. Introduction

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The purpose of this study is to investigate system-level design issues for a wireless communication system that links the central processing units (CPUs) in an advanced multi-processor computer system. The three primary requirements for the wireless communication system are that it must (1) support dynamic reconfigurability of the CPUs, (2) achieve very high data rates (on the order of 100 Gbps system capacity), and (3) achieve extremely low latencies (on the order of 10 nanoseconds). These requirements differ significantly from commercial wireless communication networks for voice and data. The objective of this effort is to identify and quantify the critical communication system design issues and trade-offs that will enable the successful achievement of the stringent requirements on dynamic reconfigurability, data rate, and latency.

In this report, we evaluate trade-offs between digital modulation formats, particularly quadrature amplitude modulation (QAM) and phase shift keying (PSK). Synchronization of the received carrier frequency and phase is an important issue for coherent demodulation. Hard-wired channels may be used to distribute a down-converted carrier, but some important questions need to be studied:

- How will phase noise in the up-converted carrier affect the bit error rate in QAM and PSK?
- What are the trade-offs in hardware complexity, carrier acquisition time, phase noise sensitivity, and overall system data transfer rate for QAM and PSK?

We address these questions in this report, which is organized as follows. In Section 2, basic facts about phase noise are reviewed. In Section 3, several receiver architectures are analyzed with respect to phase noise sensitivity and speed of carrier synchronization. These issues are critical in order to achieve extremely low latencies. Section 4 contains bit error rate (BER) performance comparisons, and examples are presented with PSK modulation, QAM, and differential demodulation. Section 5 contains concluding remarks.

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## 2. Review of Some Key Facts About Phase Noise

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Phase noise arises in the demodulated output of a digital receiver from two sources: sinusoidal oscillators contain “phase jitter,” and additive noise enters the phase synchronization loop in the receiver. The first source, oscillator phase jitter, is a random process  $\phi(t)$  that modulates the phase of a sinusoid as

$$c(t) = A_0 \cos [\omega_0 t + \phi(t)]. \quad (1)$$

The phase jitter process  $\phi(t)$  is typically modeled as zero-mean and wide-sense stationary with power spectral density (PSD)  $G_\phi(f)$ . Figure 1 illustrates the typical shape [1] of a one-sided phase jitter PSD  $G_\phi(f)$ , in which the horizontal axis represents frequency separation from  $\omega_o/(2\pi)$ . Most of the phase jitter power is near the oscillator frequency  $\omega_o/(2\pi)$ , with various rates of roll-off away from  $\omega_o/(2\pi)$ . The oscillator phase jitter is equivalently characterized by the autocorrelation

$$R_\phi(\tau) = \mathcal{F}^{-1}[G_\phi(f)], \quad (2)$$

which is the inverse Fourier transform of the PSD  $G_\phi(f)$ . The phase jitter variance

$$\sigma_\phi^2 = R_\phi(0) = \int_0^\infty G(f) df \quad (3)$$

is a commonly used statistic that is often stated as “RMS phase jitter,”  $\sigma_\phi$ .

An example of phase noise caused by additive noise entering the receiver phase synchronization loop is shown in the block diagram in Appendix A. Entering the demodulator block is a signal component  $s(t - \tau_{12})$  and an additive noise component  $n(t)$ . The demodulator output is fed back to a phase shifter that synchronizes the phase of the reference carrier with the phase of the received signal  $s(t - \tau_{12})$ . The additive noise  $n(t)$  leads to errors in adjusting the phase shifter, which causes phase noise at the demodulator output. The additive noise  $n(t)$  leads to two types of degradation: phase noise attributable to feedback control of the phase shifter, and the usual additive noise effects. The magnitude of the phase noise caused by  $n(t)$  decreases as the receiver signal-to-noise ratio (SNR) increases. However, the effects of the oscillator phase jitter in (1) are *independent* of receiver SNR.

The structure of a particular digital receiver determines the relative magnitudes of the two phase noise components at the demodulator output. For example, (4) is an approximate expression for the phase noise variance for the receiver in Appendix A. The details of (4) are explained in Section 3.1, but note the separate contributions from oscillator phase jitter and additive, white, Gaussian noise (AWGN).

The phase-locked loop (PLL) is a standard approach for synchronizing carrier frequency and phase in a receiver [1, 3]. A PLL consists of a variable frequency oscillator and a feedback loop to control the oscillator frequency. The theory of PLL operation and design is well developed, e.g., [1]–[7]. A key parameter of PLL operation is the loop bandwidth  $B_L$ , which determines the carrier acquisition time and the amount of phase noise. A larger  $B_L$  makes the PLL “fast” and enables rapid acquisition. The effect of oscillator phase jitter is reduced as  $B_L$  increases because a fast PLL tracks more of the oscillator phase jitter. However, as  $B_L$  increases, additive noise causes more phase noise. For a given scenario, an optimum loop bandwidth  $B_L$  exists that minimizes the phase noise. However, rapid carrier acquisition and low latency require  $B_L$  to be as large as possible. Thus, minimizing phase noise and maximizing acquisition speed are conflicting requirements. Minimum phase noise is desirable in order to achieve low BERs. As shown in Section 4, phase noise produces BER

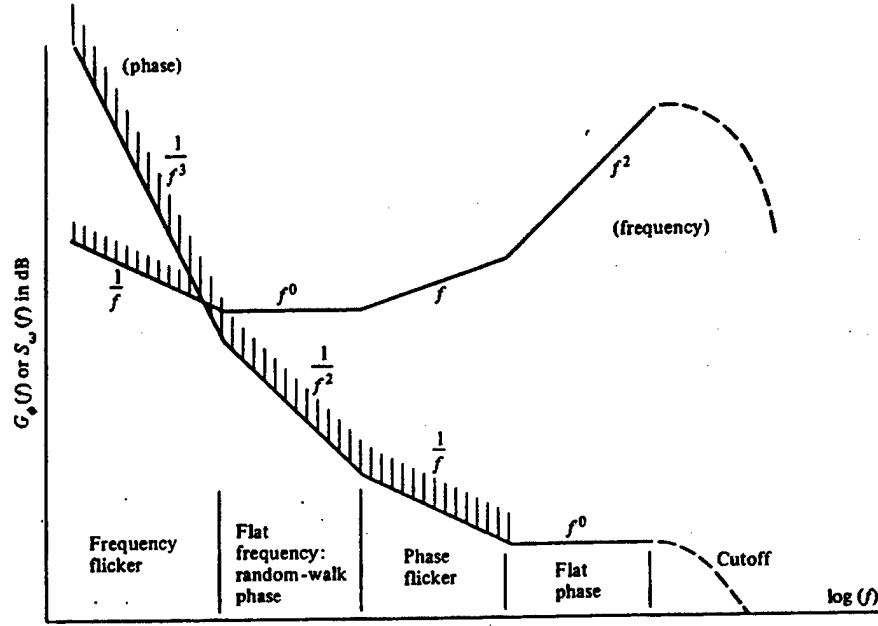


Figure 1. Typical power spectral density (PSD)  $G_\phi(f)$  of phase noise process  $\phi(t)$  in (1), from [1].

“floors” that cannot be improved with increasing SNR. For example, Figure 2d clearly shows the BER floors, where the level of the BER floor increases for larger RMS phase jitter.

In the following section, we investigate and analyze receiver structures that avoid classical PLLs in order to reduce phase noise and maximize acquisition speed. Our analysis enables comparisons of complexity/latency trade-offs between reducing phase noise and adding error-correction coding. Further, in Section 4 we compare the performance of PSK and QAM with regard to phase noise and carrier synchronization.

### 3. Architectures for Rapid Carrier Synchronization

In this section, we investigate alternatives and issues for rapid carrier synchronization with low phase noise. Some questions that we address are as follows:

- Should a reference carrier be distributed to all nodes? If so, what factors should be considered when one is choosing the frequency of the reference carrier?
- Instead of distributing a reference carrier to all nodes, should a local oscillator (LO) be generated at each node?
- What are the advantages/disadvantages of differential demodulation, with and without a reference carrier? Differential demodulation may be performed at radio frequency (RF) or at baseband.



- What can be gained with decision feedback carrier acquisition?

The main issues are whether a common reference carrier is distributed to all (transmitting and receiving) nodes and whether decision feedback (or differential) demodulation is used for phase synchronization. Several receiver structures are presented and analyzed in the remainder of this section.

### 3.1 Reference Carrier With Phase Tracking

The receiver in Appendix A has a reference oscillator that is provided by a wire connection to the transmitter (top branch) and the receiver (bottom branch). The delays  $\tau_1$  and  $\tau_2$  are determined by the length of the cables connecting the reference to the transmitter and receiver nodes. Note that the reference oscillator contains phase jitter  $\phi(t)$  as in (1). In the transmitter branch in Appendix A, the reference oscillator frequency  $\omega_o$  is multiplied by a factor  $k$  to achieve the desired carrier frequency  $k\omega_o$  for wireless transmission. This carrier is then digitally modulated with the data bits, and the signal  $s(t)$  is transmitted by an antenna. The in-phase and quadrature components of one symbol are  $A_i, B_i$ , respectively. The receiver antenna observes a noisy, delayed version of the transmitted signal,  $s(t - \tau_{12}) + n(t)$ , in which  $\tau_{12}$  is the propagation time from transmitter to receiver. The receiver demodulates the signal using a phase-shifted and frequency-multiplied version of the reference oscillator. The phase shifter is controlled by feedback from the demodulator output  $\hat{A}_i, \hat{B}_i$ . Since  $\hat{A}_i, \hat{B}_i$  are imperfect estimates of  $A_i, B_i$  attributable to the AWGN, the feedback of these values to the phase shifter produces phase noise at the demodulator output.

We have shown that the phase noise variance at demodulator output is approximately

$$\underbrace{2k^2 \left(1 + \frac{1}{L}\right) R_\phi(0) \left[1 - \frac{R_\phi(\tau_2 - \tau_1 - \tau_{12})}{R_\phi(0)}\right]}_{\text{Because of oscillator phase jitter}} + \underbrace{\frac{1}{L(2E_s/N_0)}}_{\text{Because of AWGN through phase shifter}}, \quad (4)$$

in which symbols are defined as follow:

- $\phi(t)$  is the phase jitter in the reference oscillator  $c(t) = \cos[\omega_o t + \phi(t)]$ , with PSD  $G_\phi(f)$ , autocorrelation  $R_\phi(\tau)$ , variance  $\sigma_\phi^2 = R_\phi(0)$ .
- $k$  is the multiplication factor from reference to transmitted frequency.
- $L$  is the number of past symbols used for phase estimation and adjustment of the phase shifter, in which  $L = 1$  corresponds to differential demodulation.
- $(\tau_2 - \tau_1 - \tau_{12})$  is the difference in time delay of the reference oscillator at the transmitter and receiver.
- $E_s$  is the mean energy per symbol, and  $E_b$  is the mean energy per bit. For a digital constellation with  $M$  symbols,  $E_b = E_s / \log_2 M$ .

- $N_0/2$  is the (two-sided) PSD of the AWGN,  $n(t)$ .

The component of phase noise variance in (4) attributable to oscillator phase jitter is

$$2k^2 \left(1 + \frac{1}{L}\right) \sigma_\phi^2 [1 - \rho_\phi(\tau_2 - \tau_1 - \tau_{12})] \quad (5)$$

in which we define the normalized correlation function

$$\rho_\phi(\tau) = \frac{R_\phi(\tau)}{R_\phi(0)} \in [-1, 1] \quad (6)$$

that is bounded between  $-1$  and  $+1$ . We make the following observations based on (5).

- The phase noise is reduced when the oscillator jitter is (partially) *coherent* over the time interval  $(\tau_2 - \tau_1 - \tau_{12})$ . If the propagation delays are such that  $(\tau_2 - \tau_1 - \tau_{12}) = 0$ , then  $\rho_\phi(0) = 1$  and the oscillator phase jitter is coherently cancelled. If  $\rho_\phi(\tau_2 - \tau_1 - \tau_{12}) = 0$ , then the phase jitter is incoherent and is not cancelled. Other values of  $\rho_\phi \in (-1, 0) \cup (0, 1)$  produce partial cancellation or enhancement of the phase jitter.
- The frequency multiplication factor  $k$  in Appendix A appears as a multiplicative factor in the phase noise variance (5). This fact may be exploited as follows in receiver design. Let  $\omega_c = k\omega_o$  be the desired carrier frequency. This carrier frequency can be achieved with a reference oscillator that has 10 times larger frequency  $\omega'_o = 10\omega_o$  and 10 times smaller  $k' = k/10$ , so  $\omega_c = k'\omega'_o$ . The new reference oscillator will result in less phase noise if its phase jitter variance  $\sigma_{\phi'}^2$  and normalized correlation  $\rho_{\phi'}$  satisfy

$$\sigma_{\phi'}^2 [1 - \rho_{\phi'}(\tau_2 - \tau_1 - \tau_{12})] < 100 \sigma_\phi^2 [1 - \rho_\phi(\tau_2 - \tau_1 - \tau_{12})]. \quad (7)$$

- Increasing the number of past symbols  $L$  used for phase estimation reduces the oscillator phase noise contribution by at most a factor of  $1/2$  (or 3 dB) relative to the  $L = 1$  case. However, larger  $L$  slows the speed of phase synchronization and reduces latency.
- The quantity (5) is independent of SNR (or  $E_b/N_0$ ). This fact will be evident in Section 4, such as Figure 2d, where plots of BER versus  $E_b/N_0$  show a “floor” effect that is caused by the irreducible phase error (5) attributable to oscillator jitter.

Next we consider the component of phase noise variance in (4) attributable to the AWGN that is fed back to the phase shifter,

$$\frac{1}{L(2E_s/N_0)}. \quad (8)$$

With  $L = 1$ , only one previous symbol is used for phase estimation, which is equivalent to *differential demodulation*. At high SNR, the AWGN at the receiver, denoted by  $n(t)$  in Appendix A, is well approximated by Gaussian phase noise with zero mean and variance

$1/(2E_s/N_0)$  (see, for example, [2]). Therefore at high SNR, the AWGN contributes two independent components to the phase noise, with total variance

$$\underbrace{\frac{1}{L(2E_s/N_0)}}_{\text{Phase shift estimate from } L \text{ past samples}} + \underbrace{\frac{1}{(2E_s/N_0)}}_{\text{AWGN in current sample}} = \left(\frac{1}{L} + 1\right) \frac{1}{(2E_s/N_0)}. \quad (9)$$

If the receiver has access to a perfect phase reference, such as when  $L \rightarrow \infty$ , then the first term in (9) vanishes. Note from (9) that  $L \rightarrow \infty$  reduces the noise variance by a factor of 2 relative to differential demodulation ( $L = 1$ ). Thus differential demodulation can achieve the same BER as a perfect phase reference by increasing  $E_b/N_0$  by 3 dB, as is well known. Stated another way, as  $L$  is increased, the maximum gain in  $E_b/N_0$  is 3 dB. However, larger  $L$  implies slower transient response and larger latency.

A receiver with  $L = 1$  is redrawn in Appendix B with differential demodulation at baseband instead of the phase shifter and feedback loop. We can summarize the advantages and disadvantages of differential demodulation as

1. The carrier synchronization time is minimized, thus helping to meet the goal of low latency.
2. The phase noise variance attributable to AWGN in (9) is maximum. However, the same effect as  $L \rightarrow \infty$  in (9) can be achieved by increasing  $E_b/N_0$  by 3 dB.
3. The oscillator phase jitter contribution (5) to phase noise variance is maximum for  $L = 1$ . The maximum reduction in (5) for larger  $L$  is a factor of  $1/2$  compared with the  $L = 1$  value. Thus  $L = 1$  incurs a sacrifice of 3 dB in phase noise variance because of oscillator jitter.

Differential demodulation may be attractive when the system is not power limited, thus allowing item 2 to be compensated with increased SNR. Item 3 may be mitigated if the oscillator is sufficiently coherent, i.e.,  $\rho_\phi(\tau_2 - \tau_1 - \tau_{12}) \approx 1$  in (5), so that the effect of  $L$  in (5) is negligible.

A more accurate expression for the oscillator phase jitter component (5), with  $\tau_o = \tau_2 - \tau_1 - \tau_{12}$ , is

$$2k^2\sigma_\phi^2 \left\{ \left(1 + \frac{1}{L}\right) [1 - \rho_\phi(\tau_o)] + \underbrace{\frac{2}{L^2} \sum_{i=1}^L i [\rho_\phi(iT_s - \tau_o) - 2\rho_\phi(iT_s) + \rho_\phi(iT_s + \tau_o)]}_{\text{ADDED TERM}} \right\}. \quad (10)$$

The added term (5) is zero if  $|\tau| < T_s$  and  $\rho_\phi(\cdot)$  is a triangular function.

### 3.2 Local Oscillator With Phase Tracking

The receiver in Appendix C has a local oscillator (LO) at the receiver instead of the reference oscillator in Appendix A. The phase noise variance at the demodulator output is

$$\underbrace{\left(1 + \frac{1}{L}\right) (k^2 \sigma_\phi^2 + \sigma_{\phi_2}^2)}_{\text{Because of oscillator phase jitter}} + \underbrace{\frac{1}{L(2E_s/N_0)}}_{\text{Because of AWGN through phase shifter}}, \quad (11)$$

in which  $\sigma_{\phi_2}^2$  is the variance of phase jitter in the receiver LO. In the Appendix C receiver, the independent oscillators in the transmitter and receiver make coherent cancellation of phase jitter impossible. This is evident in (11), where the first term is simply the sum of the phase jitter power in the two oscillators. This example quantifies the benefits and phase noise reduction mechanism of the receiver in Appendix A that has a common reference oscillator at the transmitter and receiver.

### 3.3 Differential Demodulation at Intermediate Frequency (IF)

The receiver in Appendix D performs differential demodulation with no reference oscillator supplied to the receiver. However, a LO is used at the receiver to convert the incoming signal to a common intermediate frequency (IF). The conversion to IF is included because a system is likely to allow multiple transmission frequencies, so a method is needed to “tune in” the desired frequency channel. Analog delay lines with length equal to one symbol period are used to perform differential demodulation of the IF signal. The carrier component in the previous symbol is used to demodulate the present symbol. The phase noise variance at the demodulator output is

$$\underbrace{4k^2 R_\phi(0) \left[1 - \frac{R_\phi(T_s)}{R_\phi(0)}\right] + 4R_{\phi,2}(0) \left[1 - \frac{R_{\phi,2}(T_s)}{R_{\phi,2}(0)}\right]}_{\text{Because of oscillator phase jitter}} + \underbrace{\frac{1}{(2E_s/N_0)}}_{\text{Because of AWGN through phase shifter}} \quad (12)$$

in which  $\phi_2(t)$  is the phase jitter in the local oscillator (LO)  $c_2(t) = \cos[\omega_c t + \phi_2(t)]$ , with PSD  $G_{\phi,2}(f)$ , autocorrelation  $R_{\phi,2}(\tau)$ , variance  $\sigma_{\phi,2}^2 = R_{\phi,2}(0)$ . The phase noise attributable to oscillator jitter is reduced if the jitter processes are (partially) coherent over the symbol period  $T_s$ . An error  $\Delta\omega$  in the IF conversion frequency leads to rotation of the quadrature axis by  $\frac{\pi \Delta\omega}{2\omega_{IF}}$  rad. Errors in the delay line lengths lead to elliptical rotation of the demodulated signals  $\hat{A}_i, \hat{B}_i$ .

Phase noise reduction occurs in (12) if the symbol period  $T_s$  is smaller than the coherence time of the phase noise processes. Therefore high data rates are beneficial to coherent phase noise reduction in differential modulation.

### 3.4 Summary of Receivers

Table 1 lists a brief summary of receiver schemes along with relevant issues regarding modulation format (QAM and PSK) and carrier synchronization. The first row refers to no

Table 1. Summary of receiver structures and issues

	QAM	PSK	Issues
No carrier	PLL or Costas Differential at RF	PLL or Costas Differential at RF	Higher complexity? Higher latency?
Transmit carrier (wireless)	Direct demod.	Direct demod.	Must strip carrier Phase matching
Reference carrier (wired)	Freq. multiply & phase sync.  Differential at baseband ?	Freq. multiply & phase sync.  Differential at baseband	Phase sync. delay Use ref. freq. > carrier freq. ? No phase sync.

reference carrier provided to the receiver, in which case, a PLL or differential demodulation is required. Disadvantages of this scheme include higher complexity and higher latency if the PLL is used. If the carrier is transmitted as a wireless signal, then this carrier must be stripped and phase matched. If a reference carrier is provided to the receiver through a wire, then frequency multiplication and phase matching are required. The reference frequency may be chosen to minimize phase noise. Differential demodulation may be performed at baseband more simply for PSK than for QAM [3].

#### 4. Performance Comparisons

In this section, numerical examples are presented to illustrate the effects of phase noise on the BER. We consider  $M$ -ary PSK and QAM for various values of  $M$ , and we also demonstrate the impact of differential demodulation. Appendix E contains definitions for the PSK and QAM constellations and an outline of the procedure used to approximate the BER with phase noise. The phase noise is modeled as independent and identically distributed (iid) in each symbol, with Gaussian distribution, zero mean, and variance as described in the next paragraph. The BER also includes the effects of the AWGN in the received signals, as described in Appendix E.

The BER results in this section are presented to show the effects of the two components in the phase noise variance expressions (4), (11), (12), i.e., “Because of oscillator phase jitter” and “Because of AWGN through phase shifter”. The oscillator phase jitter component will be specified as RMS PHASE JITTER, which is the square root of the first term in (4), (11), (12). Thus the examples in this section apply to all of the receivers presented in Section 3, but for simplicity we orient our discussion toward (4). The receivers differ only in the system parameters that determine the RMS PHASE JITTER. The “Because of AWGN

through phase shifter" term in (4), (11), (12) will be included since we will show plots of BER versus  $E_b/N_0$ . One application of the framework presented in this section is the potential to perform a trade-off analysis between alternatives for improving the BER. For example, we can improve the BER by reducing phase noise or adding forward error correction (FEC) coding. The results in this section quantify how much reduction in phase noise is required in order to achieve a desired BER. The complexity of phase noise reduction may be compared with the complexity and latency of FEC to determine which is best to achieve the system objectives.

The performance of  $M$ -PSK for various  $M$  with phase jitter is shown in Figures 2 and 3, from which we observe the following.

- BPSK ( $M = 2$ ) and QPSK ( $M = 4$ ) are resistant to a moderate amount of phase noise (RMS phase error  $< 4^\circ$ ).
- The BER "floor" attributable to phase jitter is evident for  $M$ -ary PSK for  $M \geq 8$ . The floor appears when the constellation becomes dense enough so that the oscillator jitter causes symbol errors, independent of SNR.
- 32- and 64-ary PSK are significantly affected by phase jitter, even with RMS phase jitter of  $1^\circ$  or  $2^\circ$ .
- 16-PSK has reasonable BER performance if the RMS phase jitter is less than  $2^\circ$ . For example, to achieve  $10^{-6}$  BER, the  $(E_b/N_0)$  with no phase jitter is 18 dB, while with  $2^\circ$  RMS phase jitter, the required  $(E_b/N_0)$  is 23 dB. So  $2^\circ$  RMS phase jitter costs 5 dB in SNR.

Figures 4a and 4b show plots of BER performance for 16 PSK and 16 QAM with the phase noise variance given by (4). The "PERFECT PHASE REF." corresponds to an oscillator at the receiver that is perfectly coherent with the transmitter. The "RMS PHASE JITTER" refers to the square root of the first term in (4), so it is the contribution attributable to the oscillator phase jitter. Note that the first term in (4) includes the combined effects of frequency multiplication by  $k$  and coherence of the oscillator phase jitter. The "RMS PHASE JITTER = 0" corresponds to the effect of the second term in (4) by itself, i.e., the effect of receiver AWGN on the phase noise and BER. The plots in Figures 4a and 4b are for phase estimation based on  $L = 1$  past symbols. Note that the "RMS PHASE JITTER = 0" curve is shifted from the "PERFECT PHASE REF." curve by 3 dB, which is consistent with the 3 dB SNR penalty of differential modulation discussed with reference to (9). This case is equivalent to differential modulation because the phase is estimated from only one previous symbol. Phase noise attributable to oscillator phase jitter further degrades the BER performance. As shown in Figures 4c and 4d, we can reduce the effect of AWGN by averaging more past symbols  $L > 1$ , but this produces a longer synchronization time. However, if the phase is not expected to change quickly, as in a system with fixed transmitter/receiver locations, then a larger  $L$  may be beneficial, with a maximum gain in

SNR of 3 dB. On the other hand, if the system is not power limited and the 3 dB of additional SNR are available, then the receiver with  $L = 1$  is simpler. Note from the first term in (4) that averaging over more past symbols  $L$  reduces the first term by at most 50%.

Figures 4c and 4d present results for  $L \rightarrow \infty$  in (4), so the AWGN contribution to the phase noise through the phase shifter vanishes. In this case, NO PHASE JITTER is identical to PERFECT PHASE REF. The curves in Figures 4c and 4d are generally shifted 3 dB to the left with respect to Figures 4a and 4b, except for the floors, which are identical in the PSK pairs (Figures 4a, 4c) and the QAM pairs (Figures 4b, 4d). Note that with zero phase noise, 16-PSK requires 4 dB more  $\frac{E_b}{N_0}$  than 16-QAM. Also, 16-PSK and 16-QAM are tolerant of RMS phase jitter  $< 1^\circ$ , but 16-PSK is more sensitive to phase jitter and has higher BER floors. A reduction in RMS phase error of  $1^\circ$  reduces the BER by several orders of magnitude.

Figure 5 shows plots of the same data in Figures 4c and 4d in another form, and we add the following observations.

- Figure 4 illustrates that QAM is more tolerant of the additive noise and the phase jitter, as expected. The BER floor is significantly lower for QAM compared with PSK.
- Figure 4 also shows that a reduction in RMS phase error of  $1^\circ$  reduces the BER floor by several orders of magnitude. For a given RMS phase error, forward error correction (FEC) coding is an alternate way to reduce the BER floor. Increased hardware complexity and latency are required to reduce the RMS phase error and to implement FEC coding. A designer should carefully evaluate the complexity of reducing RMS phase error against the complexity of FEC coding in order to achieve a desired BER.
- From Figure 5(b),  $2^\circ$  RMS phase jitter costs about 1.5 dB in SNR for 16 QAM at  $10^{-6}$  BER, while the cost is 5 dB for 16 PSK.
- Figure 5(a) illustrates that  $1^\circ$  RMS phase jitter causes little degradation to both 16 QAM and 16 PSK. Therefore, if  $1^\circ$  RMS phase jitter can be achieved, then 16 PSK may be a viable alternative to 16 QAM. An advantage of 16 PSK is its constant modulus, so automatic gain control is not needed, and it is resistant to nonlinearities in amplifiers or other components. A disadvantage of PSK is the increased  $(E_b/N_0)$ . At  $10^{-6}$  BER, 16 PSK requires about 4.5 dB greater  $(E_b/N_0)$  compared with 16 QAM.

We conclude this section with some general remarks regarding PSK versus QAM.

- PSK (constant modulus)
  - Insensitive to nonlinear amplification
  - Better for equalization (if needed)
  - Not automatic gain control needed

- More sensitive to phase noise
- QAM
  - Sensitive to nonlinearities
  - Worse for equalization
  - Less sensitive to phase noise
  - More complex implementation
  - Achieves more bits/Hz for a given SNR and phase noise

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## 5. Concluding Remarks

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We have presented an analysis of phase noise sensitivity for several digital receiver architectures. The speed of carrier synchronization has also been assessed for the receivers. Quantitative comparisons between PSK and QAM were performed as a function of SNR and RMS phase jitter to aid in the design of a wireless communication system for connection of CPUs in a multi-processor computer.



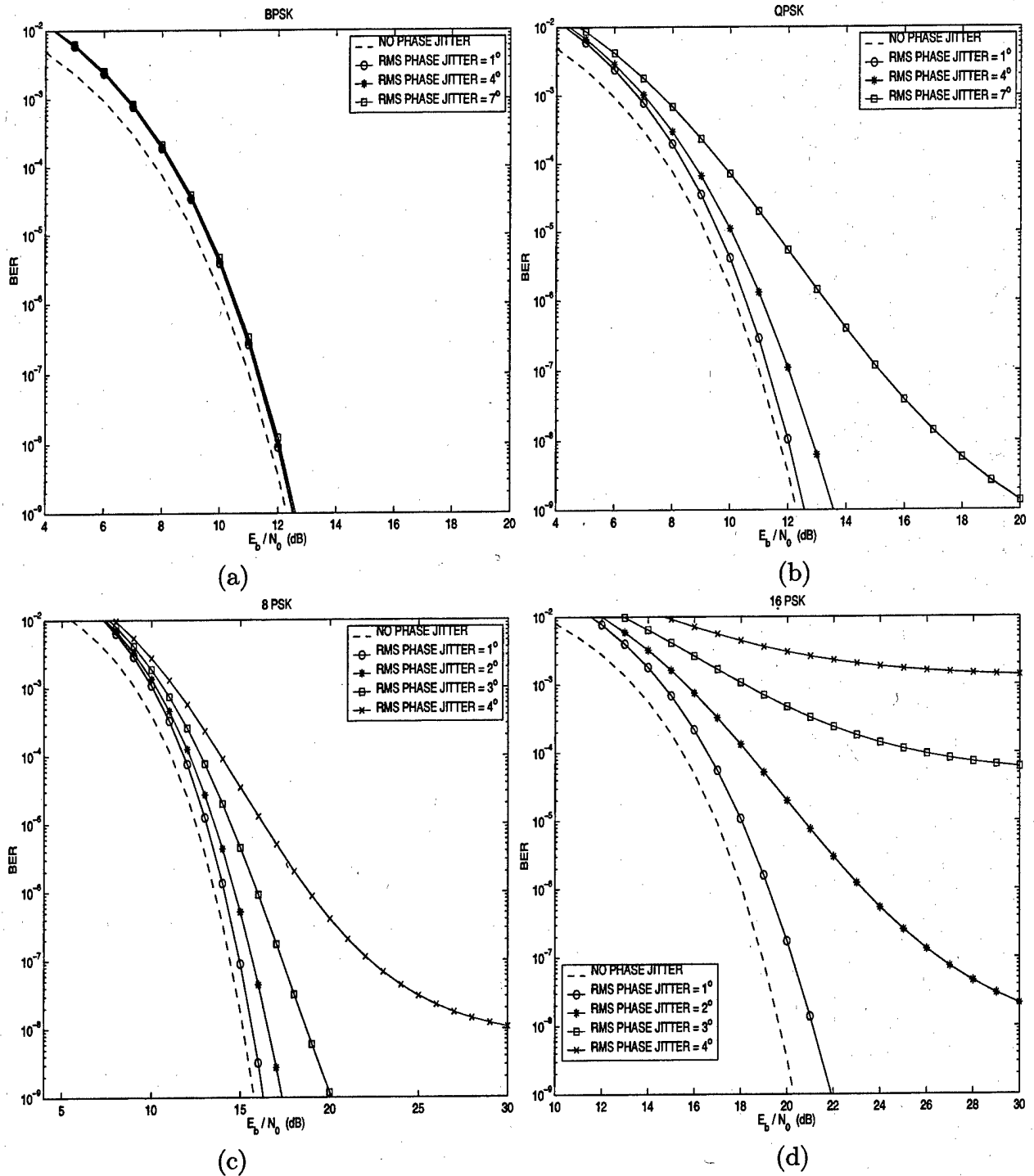


Figure 2. Effect of phase jitter on  $M$ -ary PSK for (a)  $M = 2$ , (b)  $M = 4$ , (c)  $M = 8$ , (d)  $M = 16$ .

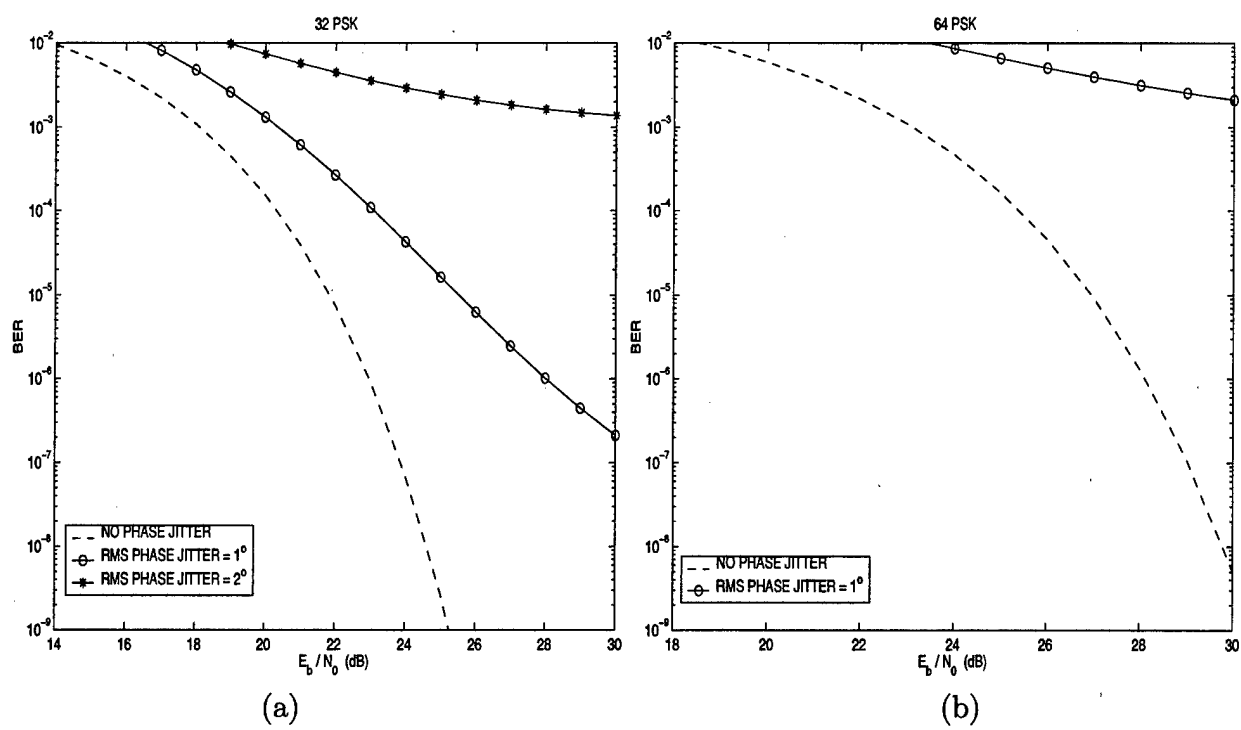


Figure 3. Effect of phase jitter on  $M$ -ary PSK for (a)  $M = 32$  and (b)  $M = 64$ .

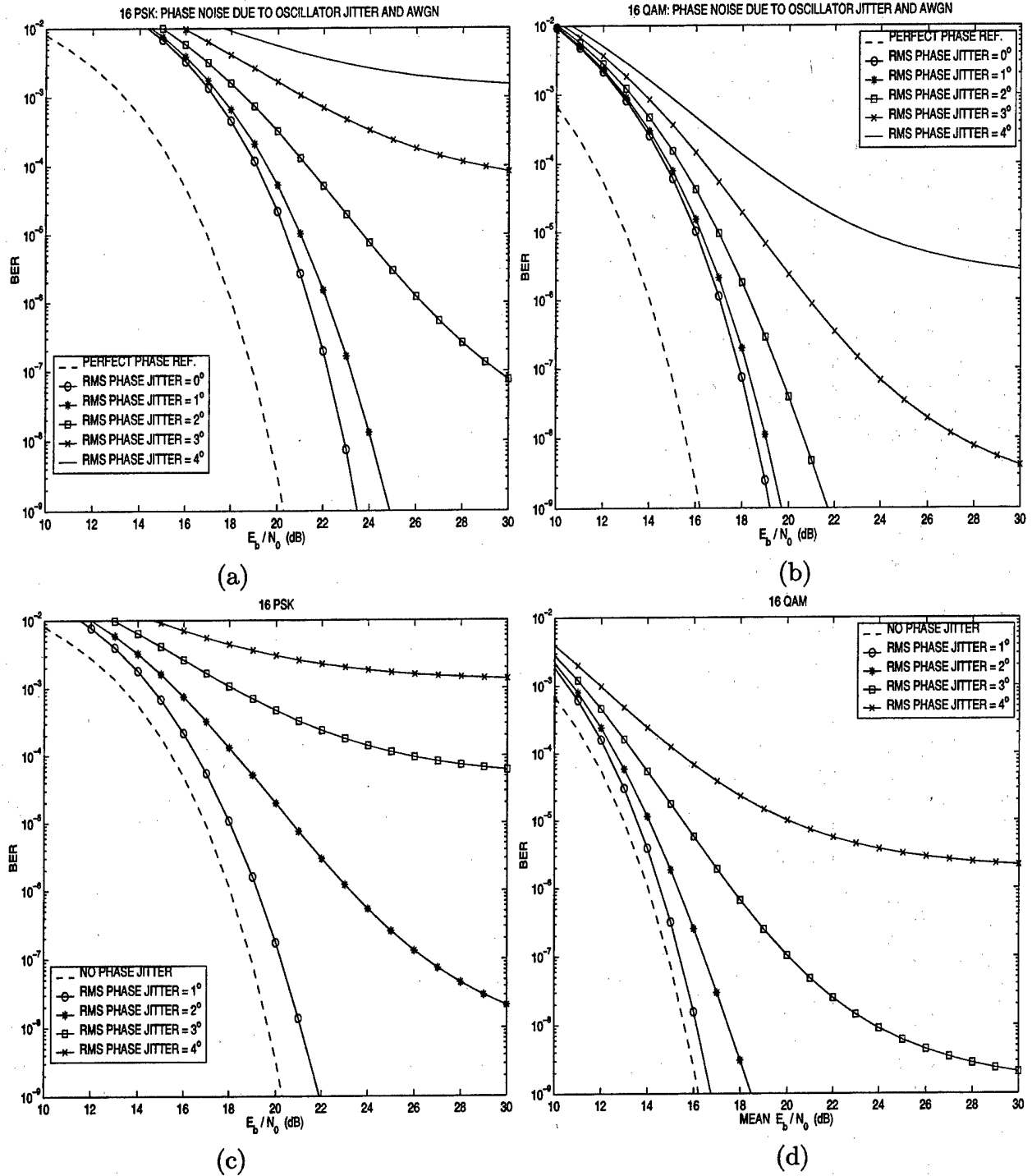


Figure 4. (a) 16 PSK and (b) 16 QAM with phase noise components attributable to oscillator jitter and AWGN as in (4) with  $L = 1$ . (c) 16 PSK and (d) 16 QAM with phase noise components attributable to oscillator jitter and AWGN as in (4) with  $L \rightarrow \infty$ .

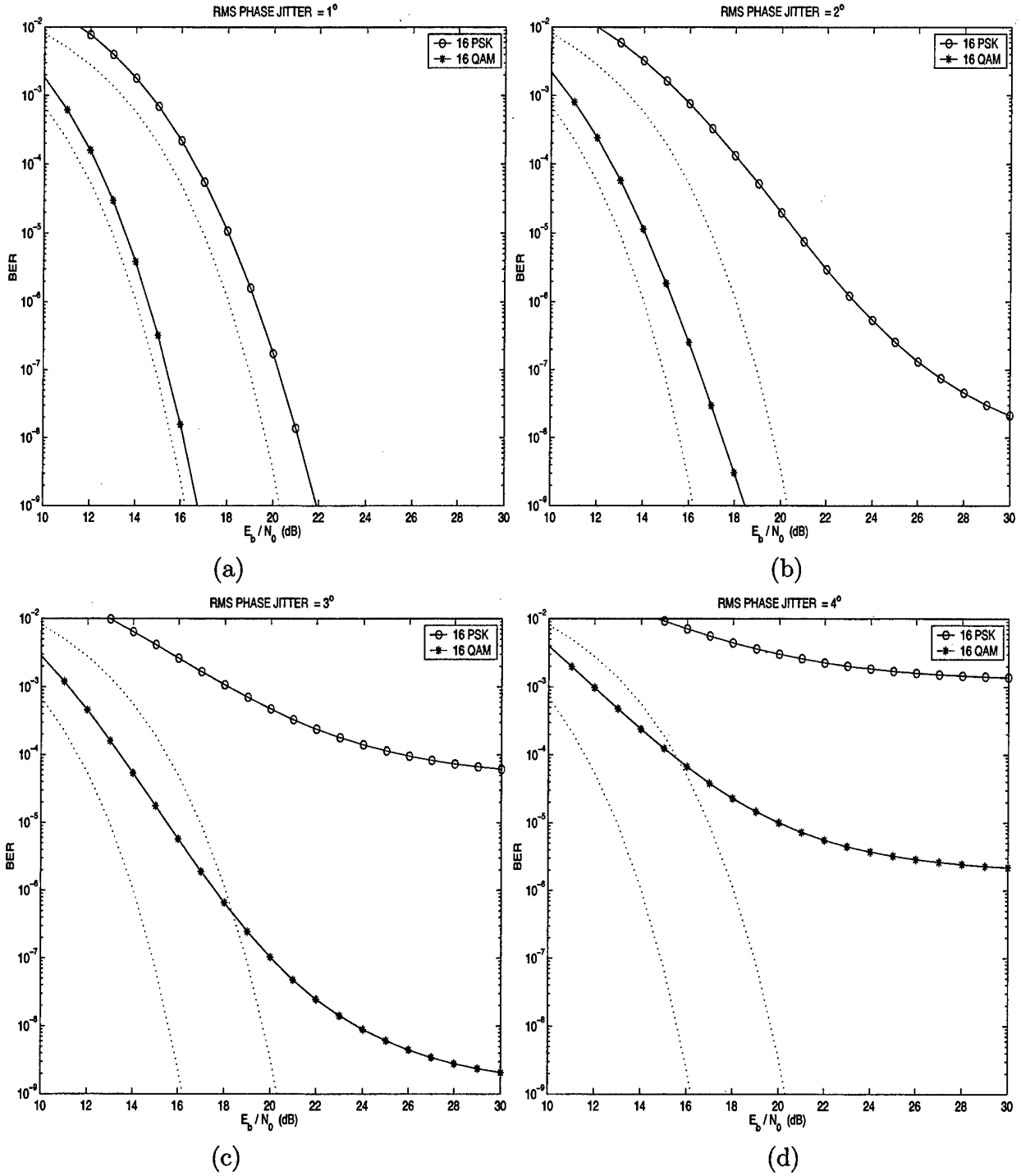
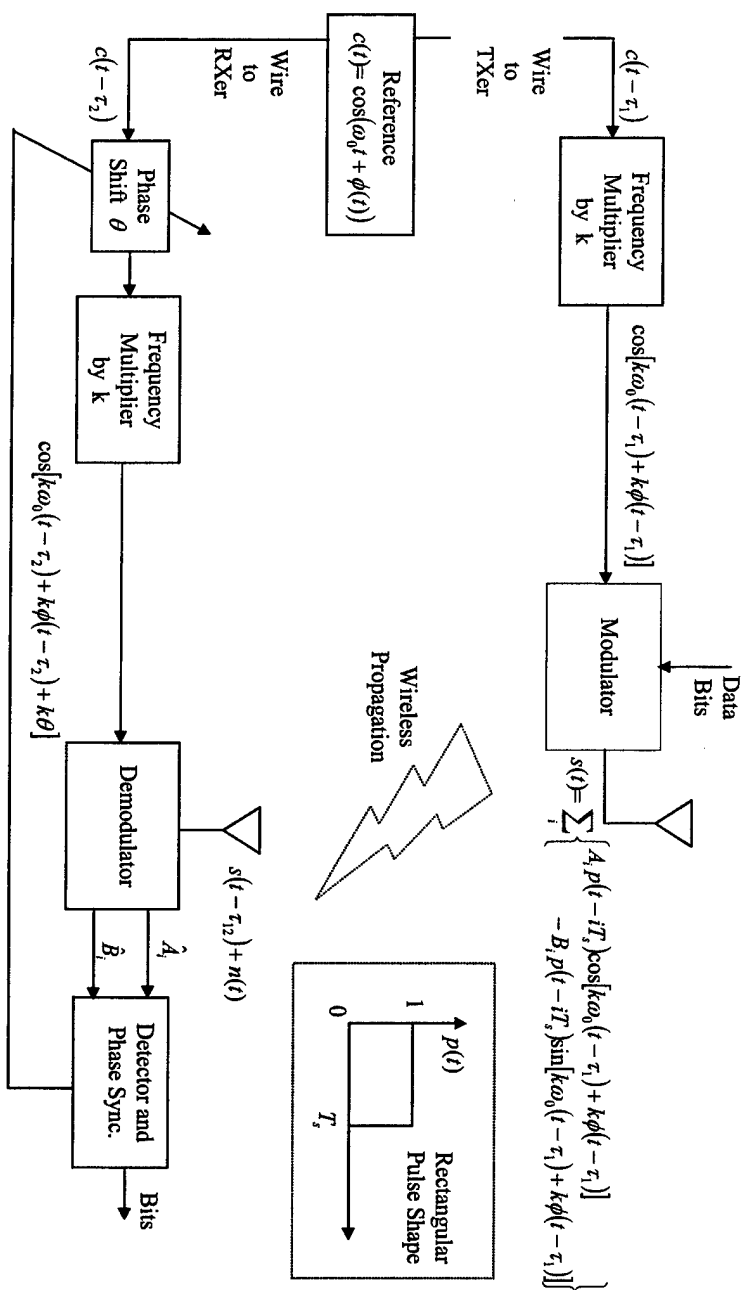


Figure 5. Comparison of 16 PSK and 16 QAM with various amounts of phase jitter, where the dashed lines correspond to no phase jitter. RMS phase jitter is (a)  $1^\circ$ , (b)  $2^\circ$ , (c)  $3^\circ$ , and (d)  $4^\circ$ . In each case, the PSK curve lies *above* the corresponding QAM curve. These plots are for the case  $L \rightarrow \infty$  and display the same data as in Figures 4c and d.

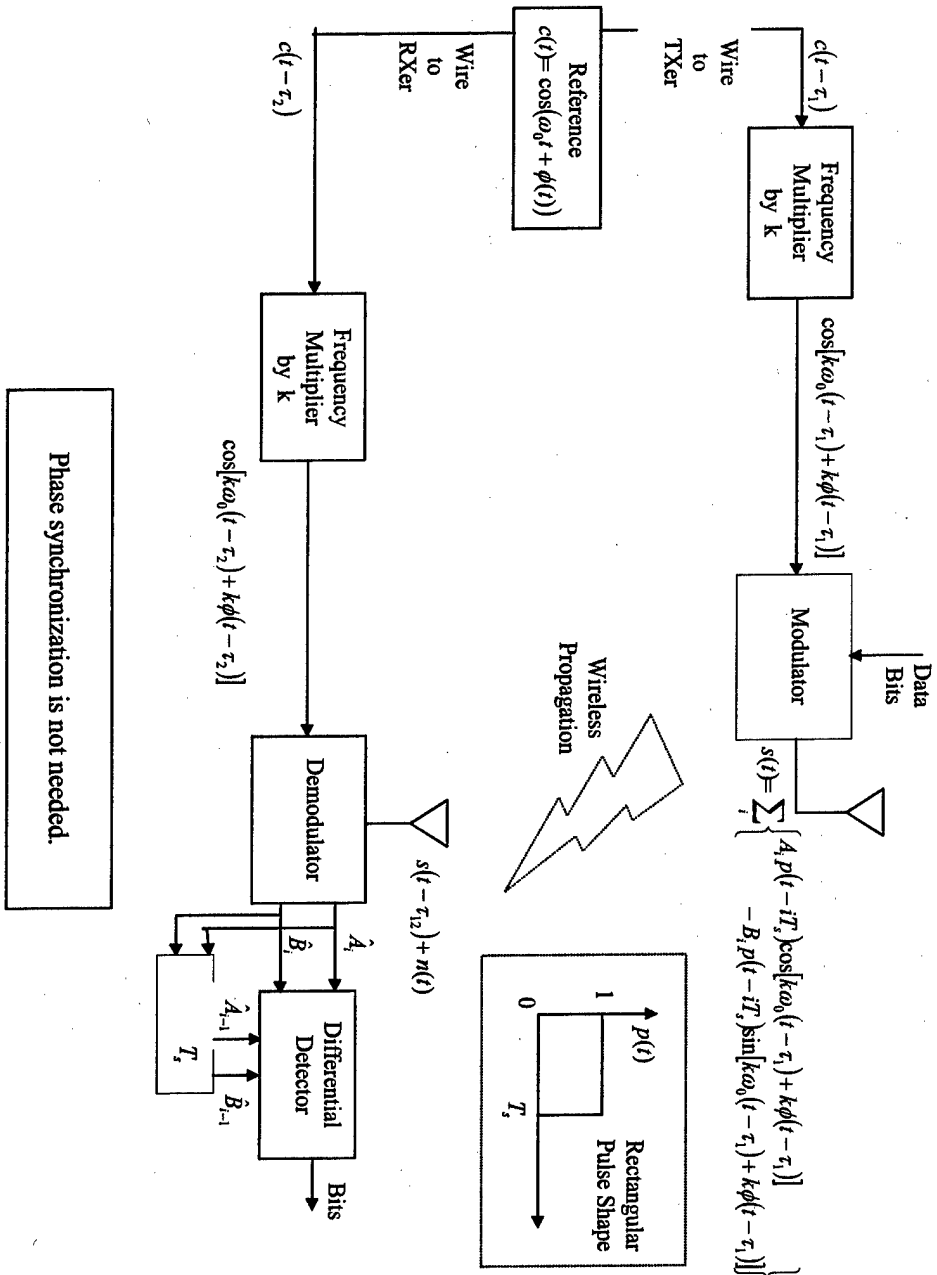
## **Appendices**

Appendices A through E are presented on the following pages.

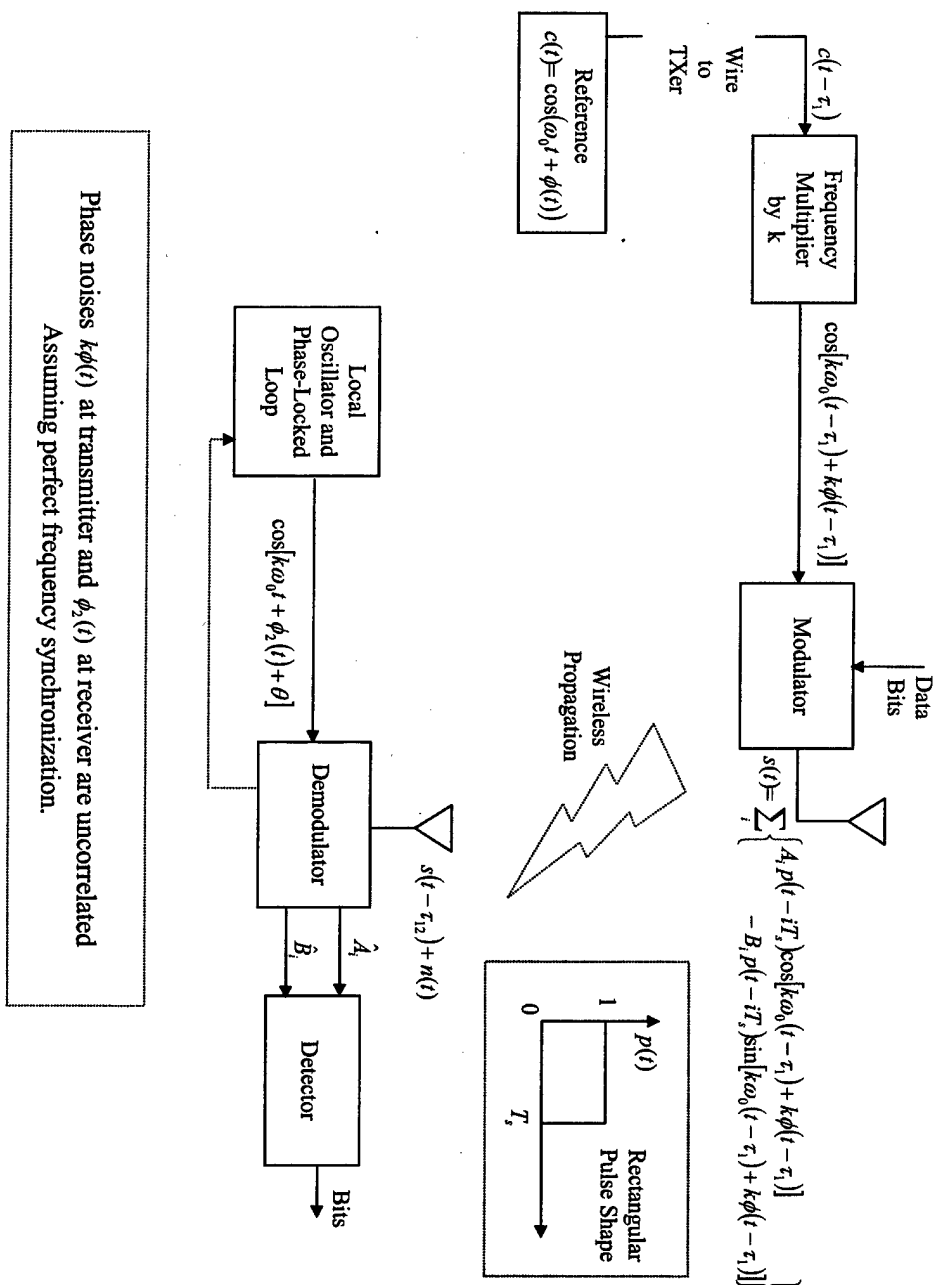
## A Reference Carrier With Phase Tracking



## B Differential Demod. at Baseband With Reference Carrier

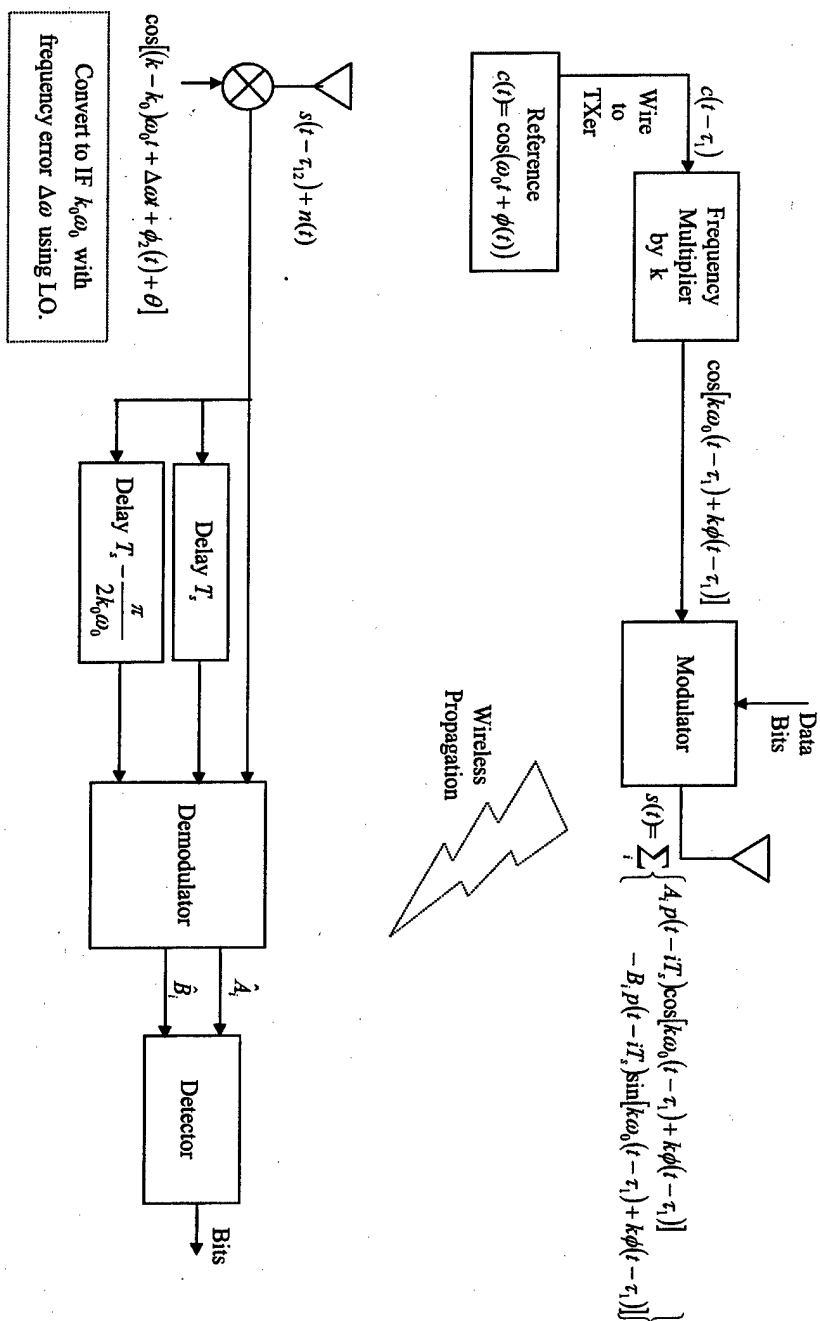


## C Local Oscillator With Phase Tracking





## D Differential Demodulation at Intermediate Frequency



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## E BER Approximations With Phase Noise

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The  $M$ -ary PSK constellation has signals that lie on a circle with radius  $\sqrt{E_s}$ , with signals equally spaced with angle  $(2\pi)/M$  between adjacent signals. The  $M$ -ary QAM constellation has signals on a square grid with in-phase and quadrature components  $\pm a, \pm 3a, \dots, \pm(\sqrt{M}-1)a$ , in which  $a = \sqrt{(3E_s)/[2(M-1)]}$ .

The BER with phase noise is computed in the standard way [1, 4, 5, 8], which we briefly review. Let  $(A_i, B_i)$  be the true in-phase (I) and quadrature (Q) components for symbol  $i$ , and suppose that  $(A_i, B_i)$  are observed with phase error  $\phi$  and AWGN  $(n_1, n_2)$ . The phase error leads to a rotation, so the  $(I, Q)$  observation with phase error and AWGN is

$$\mathbf{Z} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} A_i \\ B_i \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \mathbf{R}_\phi \mathbf{s}_i + \mathbf{n}. \quad (13)$$

The pdf of  $\mathbf{Z}$ , conditioned on phase error  $\phi$  and signal  $i$ , is

$$f(\mathbf{Z}|\phi, \mathbf{s}_i) = \frac{1}{\pi N_0} \exp \left[ -\frac{1}{N_0} \|\mathbf{Z} - \mathbf{R}_\phi \mathbf{s}_i\|^2 \right]. \quad (14)$$

We model the phase noise  $\phi$  as a Gaussian random variable with variance  $\sigma_\phi^2$ , so

$$f_\phi(\phi) = \frac{1}{\sqrt{2\pi}\sigma_\phi} \exp \left( -\frac{\phi^2}{2\sigma_\phi^2} \right). \quad (15)$$

Then the pdf of  $\mathbf{Z}$  given signal  $i$  is

$$f(\mathbf{Z}|\mathbf{s}_i) = \int_{-\pi}^{\pi} \frac{1}{\pi N_0} \exp \left[ -\frac{1}{N_0} \|\mathbf{Z} - \mathbf{R}_\phi \mathbf{s}_i\|^2 \right] f_\phi(\phi) d\phi. \quad (16)$$

If  $\mathcal{R}_i$  is the decision region for signal  $i$ , the probability of symbol error given that symbol  $i$  is transmitted is

$$P(\varepsilon|\mathbf{s}_i) = \int_{\mathbf{Z} \notin \mathcal{R}_i} f(\mathbf{Z}|\mathbf{s}_i) d\mathbf{Z}. \quad (17)$$

We evaluate (17) with decision regions that correspond to choosing the symbol that is closest to the observation, which is the maximum likelihood classifier. Then, assuming that nearest neighbor symbols are encoded in binary with a Gray code, the probability of bit error (or BER) is obtained as  $P_b(\varepsilon|\mathbf{s}_i) = P(\varepsilon|\mathbf{s}_i)/\log_2(M)$ .

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